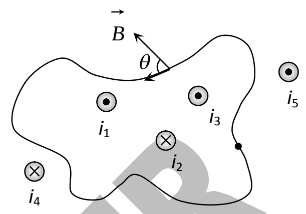
MAGNETIC EFFECTS OF CURRENT-2
[Ampere's Circuital Law and its applications]

Ampere's Law
Amperes law gives another method to calculate the magnetic field due to a given current distribution.

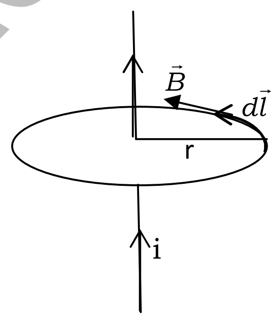
Line integral of the magnetic field $\vec{B}$ around any closed curve is equal to μ_0 times the net current i threading through the area enclosed by the curve *i.e.*



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \sum i = \mu_0(i_1 + i_3 - i_2)$$

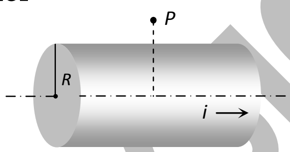
Total current crossing the above area is $(i_1 + i_3 - i_2)$. Any current outside the area is not included in net current. (Outward $\odot \rightarrow +ve$, Inward $\otimes \rightarrow -ve$)

Justification
Let us consider a very long wire carrying current i, such that magnetic field at a perpendicular distance r from the wire is given by $B = \dfrac{\mu_o i}{2\pi r}$. This field is constant in magnitude at all the points a distance r around the wire. If we join all these points we get circular field line such that tangent to it at any point gives the direction of magnetic field.
If we take the line integral of this magnetic field over this closed loop, we get

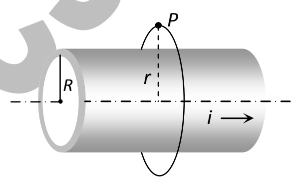


$$\oint \vec{B}.d\vec{l} = \oint B dl \cos 0^0 = B\oint dl = B\, 2\pi r = \frac{\mu_o i}{2\pi r} 2\pi r = \mu_o i$$

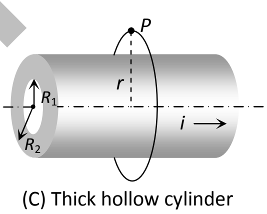
Magnetic Field Due to a Cylindrical Wire
Magnetic field due to a cylindrical wire is obtained by the application of Ampere's law
(1) **Outside the cylinder**



(A) Solid cylinder



(B) Thin hollow cylinder

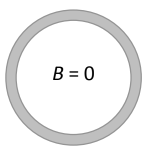


(C) Thick hollow cylinder

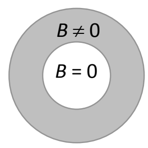
In all above cases magnetic field outside the wire at P $\oint \vec{B}.\vec{dl} = \mu_0 i \Rightarrow B\int dl = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 i \Rightarrow$

$B_{out} = \dfrac{\mu_0 i}{2\pi r}$. In all the above cases $B_{surface} = \dfrac{\mu_0 i}{2\pi R}$

(2) **Inside the hollow cylinder :** Magnetic field inside the hollow cylinder is zero.

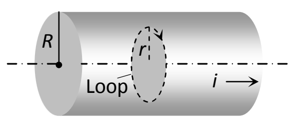


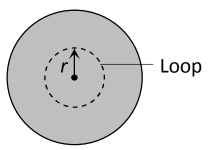
(A) Thin hollow cylinder



(B) Thick hollow cylinder

(3) **Inside the solid cylinder :** Current enclosed by loop (i') is lesser then the total current (i)

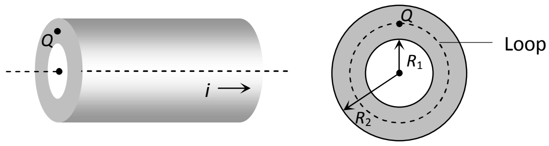




Current density is uniform *i.e.* $J = J' \Rightarrow \; i' = i \times \dfrac{A'}{A} = i\left(\dfrac{r^2}{R^2}\right)$

Hence at inside point $\oint \overrightarrow{B_{in}} . \overrightarrow{dl} = \mu_0 i' \Rightarrow \; B = \dfrac{\mu_0}{2\pi} \cdot \dfrac{ir}{R^2}$

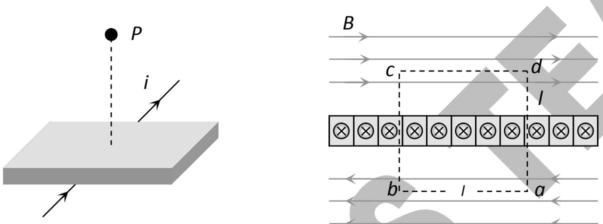
(4) **Inside the thick portion of hollow cylinder:** Current enclosed by loop is given as $i' = i \times \dfrac{A'}{A} = i \times \dfrac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$



Hence at point Q $\oint \vec{B} . \overrightarrow{dl} = \mu_0 i' \Rightarrow B = \dfrac{\mu_0 i}{2\pi r} \cdot \dfrac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$

Magnetic Field Due to an Infinite Sheet Carrying Current

The figure shows an infinite sheet of current with linear current density j (A/m). Due to symmetry the field line pattern above and below the sheet is uniform. Consider a square loop of side l as shown in the figure.



$\int_a^b \vec{B}.\overrightarrow{dl} + \int_b^c \vec{B}.\overrightarrow{dl} + \int_c^d \vec{B}.\overrightarrow{dl} + \int_d^a \vec{B}.\overrightarrow{dl} = \mu_0 i$ (By Ampere's law)

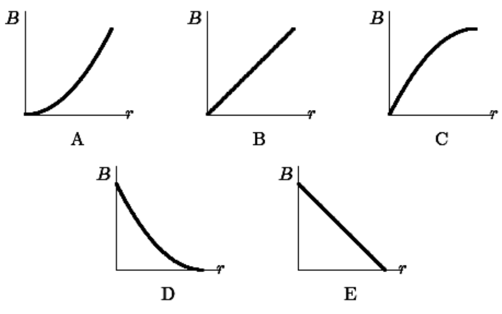
Since $B \perp dl$ along the path $b \to c$ and $d \to a$, therefore, $\int_b^c \vec{B}.\overrightarrow{dl} = 0$; $\int_d^a \vec{B}.\overrightarrow{dl} = 0$

Also, $B \,||\, dl$ along the path $a \to b$ and $c \to d$, thus $\int_a^b \vec{B}.\overrightarrow{dl} + \int_d^a \vec{B}.\overrightarrow{dl} = 2Bl$

The current enclosed by the loop is $i = jl$. Therefore, according to Ampere's law $2Bl = \mu_0 (jl)$ or $B = \dfrac{\mu_0 j}{2}$

Understanding Concept:-

1. Which graph correctly gives the magnitude of the magnetic field outside an infinitely long straight current-carrying wire as a function of the distance r from the wire?



2. In Ampere's law, $\oint \vec{B}.d\vec{s} = \mu_o i$ the integration must be over any:

A. surface B. closed surface
C. path D. closed path
E. closed path that surrounds all the current producing $\vec{B}$

3. In Ampere's law, $\oint \vec{B}.d\vec{s} = \mu_o i$ the symbol $d\vec{s}$ is:

A. an infinitesimal piece of the wire that carries current i
B. in the direction of $\vec{B}$
C. perpendicular to $\vec{B}$
D. a vector whose magnitude is the length of the wire that carries current i
E. none of the above

4. In Ampere's law, $\oint \vec{B}.d\vec{s} = \mu_o i$ the direction of the integration around the path:

A. must be clockwise
B. must be counterclockwise

C. must be such as to follow the magnetic field lines
D. must be along the wire in the direction of the current
E. none of the above

5. A long straight wire carrying a 3.0A current enters a room through a window 1.5m high and 1.0m wide. The path integral $\oint \vec{B}.d\vec{s}$ around the window frame has the value (in T·m):

A. 0.20 B. 2.5×10^{-7} C. 3.0×10^{-7} D. 3.8×10^{-6} E. none of these

6. Two long straight wires enter a room through a door. One carries a current of 3.0A into the room while the other carries a current of 5.0A out. The magnitude of the path integral $\oint \vec{B}.d\vec{s}$ around the door frame is:

A. 2.5×10^{-6} T · m B. 3.8×10^{-6} T · m C. 6.3×10^{-6} T · m D. 1.0×10^{-5} T · m
E. none of these

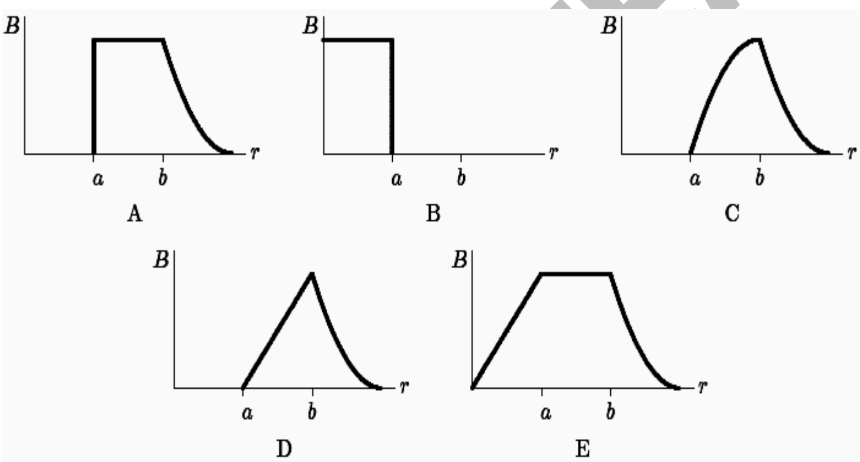
7. If the magnetic field $\vec{B}$ is uniform over the area bounded by a circle with radius R, the net current through the circle is:

A. 0 B. $2\pi RB/\mu_0$ C. $\pi R^2B/\mu_0$ D. $RB/2\mu_0$ E. $2RB/\mu_0$

8. The magnetic field at any point is given by $\vec{B} = A\vec{r} \times \hat{k}$ where $\vec{r}$ is the position vector of the point and A is a constant. The net current through a circle of radius R, in the xy plane and centered at the origin is given by:

A. $\pi AR^2/\mu_0$ B. $2\pi AR/\mu_0$ C. $4\pi AR^3/3\mu_0$ D. $2\pi AR^2/\mu_0$ E. $\pi AR^2/2\mu_0$

9. A hollow cylindrical conductor (inner radius = a, outer radius = b) carries a current i uniformly spread over its cross section. Which graph below correctly gives B as a function of the distance r from the center of the cylinder?



10. A long straight cylindrical shell carries current i parallel to its axis and uniformly distributed over its cross section. The magnitude of the magnetic field is greatest:

A. at the inner surface of the shell
B. at the outer surface of the shell
C. inside the shell near the middle
D. in hollow region near the inner surface of the shell
E. near the center of the hollow region

11. A long straight cylindrical shell has inner radius Ri and outer radius Ro. It carries current i, uniformly distributed over its cross section. A wire is parallel to the cylinder axis, in the hollow region (r < Ri). The magnetic field is zero everywhere outside the shell (r > Ro). We conclude that the wire:

A. is on the cylinder axis and carries current i in the same direction as the current in the shell
B. may be anywhere in the hollow region but must be carrying current i in the direction opposite to that of the current in the shell
C. may be anywhere in the hollow region but must be carrying current i in the same direction as the current in the shell
D. is on the cylinder axis and carries current i in the direction opposite to that of the current in the shell
E. does not carry any current

12. A long straight cylindrical shell has inner radius Ri and outer radius Ro. It carries a current i, uniformly distributed over its cross section. A wire is parallel to the cylinder axis, in the hollow region (r < Ri). The magnetic field is zero everywhere in the hollow region. We conclude that the wire:

A. is on the cylinder axis and carries current i in the same direction as the current in the shell

B. may be anywhere in the hollow region but must be carrying current i in the direction opposite to that of the current in the shell

C. may be anywhere in the hollow region but must be carrying current i in the same direction as the current in the shell

D. is on the cylinder axis and carries current i in the direction opposite to that of the current in the shell

E. does not carry any current

13. A long, straight wire of radius R carries a current distributed uniformly over its cross-section. The magnitude of the magnetic field is

(A) maximum at the axis of the wire
(B*) minimum at the axis of the wire
(C*) maximum at the surface of the wire
(D) minimum at the surface of the wire.

14. A hollow tube is carrying an electric current along its length distributed uniformly over its surface. The magnetic field

(A) increases linearly from the axis to the surface
(B*) is constant inside the tube
(C*) is zero at the axis
(D) is zero just outside the tube

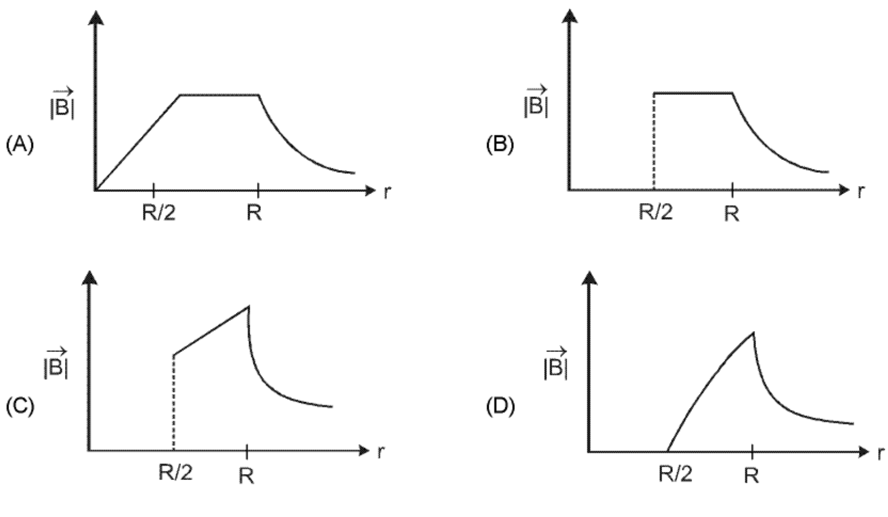
15. In a coaxial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero.

(A*) outside the cable
(B) inside the inner conductor
(C) inside the outer conductor
(D) in between the two conductors.

16. A steady electric current is flowing through a cylindrical conductor.

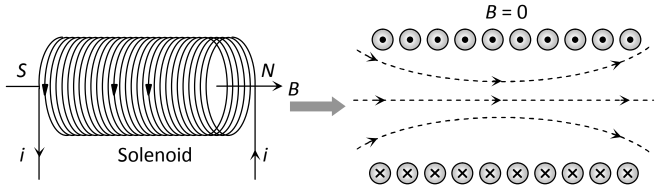
(A) the electric field at the axis of the conductor is zero
(B*) the magnetic field at the axis of the conductor is zero
(C*) the electric field in the vicinity of the conductor is zero
(D) the magnetic field in the vicinity of the conductor is zero

17. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field B as a function of the radial distance r from the axis is given by: **[IIT- 2012]**



Solenoid

A cylinderical coil of many tightly wound turns of insulated wire with generally diameter of the coil smaller than its length is called a solenoid.

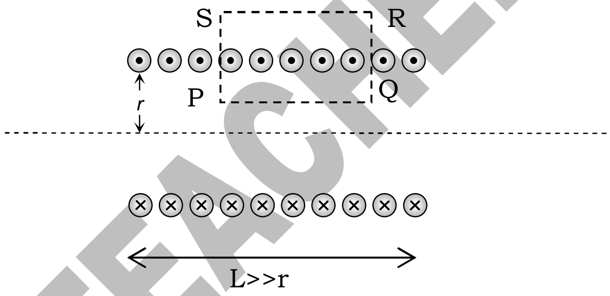


A magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of solenoid.

Calculating the field of a long Solenoid using Ampere's law

Let us consider a long solenoid of length L>>r, carrying current i, having N as number of turns.

Now if we consider the loop PQRS and apply Ampere's law we get

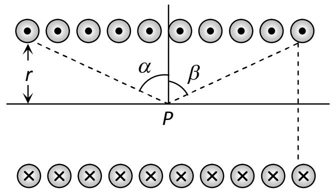


$$\oint \vec{B}.d\vec{l} = \int_P^Q \vec{B}.d\vec{l} + \int_Q^R \vec{B}.d\vec{l} + \int_R^S \vec{B}.d\vec{l} + \int_S^P \vec{B}.d\vec{l}$$

$$= B(PQ) + 0 + 0 + 0 = \mu_0\, n'i$$

$$\therefore B = \frac{\mu_0\, n'i}{PQ} = \mu_0\, n\, i$$

(1) **Finite length solenoid :** If N = total number of turns, l = length of the solenoid, n = number of turns per unit length $= \frac{N}{l}$



(i) Magnetic field inside the solenoid at point P is given by $B = \frac{\mu_0}{4\pi}(2\pi ni)[\sin\alpha + \sin\beta]$

(ii) **Infinite length solenoid :** If the solenoid is of infinite length and the point is well inside the solenoid *i.e.* $\alpha = \beta = (\pi/2)$.

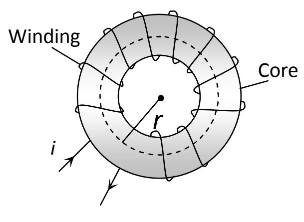
So $\quad B_{in} = \mu_0 ni$

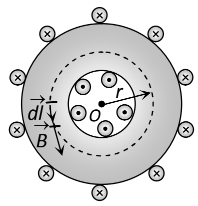
(iii) If the solenoid is of infinite length and the point is near one end *i.e.* $\alpha = 0$ and $\beta = (\pi/2)$ so

$$B_{end} = \frac{1}{2}(\mu_0 ni) \qquad \left(B_{end} = \frac{1}{2}B_{in}\right)$$

Toroid

A toroid can be considered as a ring shaped closed solenoid. Hence it is like an endless cylindrical solenoid.





Consider a toroid having n turns per unit length. Magnetic field at a point P in the figure is given as $B = \frac{\mu_0 Ni}{2\pi r} = \mu_o ni$ where $n = \frac{N}{2\pi r}$

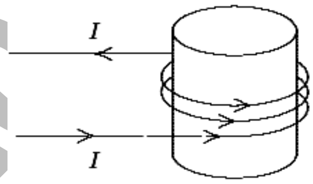
In an ideal toroid the coils are circular. In reality the turns of the toroidal coil form a helix and there is always a small magnetic field external to the toroid.

Understanding Concept:-

1. The magnetic field B inside a long ideal solenoid is independent of:
A. the current
B. the core material
C. the spacing of the windings
D. the cross-sectional area of the solenoid
E. the direction of the current

2. Two long ideal solenoids (with radii 20mm and 30 mm, respectively) have the same number of turns of wire per unit length. The smaller solenoid is mounted inside the larger, along a common axis. The magnetic field within the inner solenoid is zero. The current in the inner solenoid must be:
A. two-thirds the current in the outer solenoid
B. one-third the current in the outer solenoid
C. twice the current in the outer solenoid
D. half the current in the outer solenoid
E. the same as the current in the outer solenoid

3. Magnetic field lines inside the solenoid shown are:



A. clockwise circles as one looks down the axis from the top of the page
B. counterclockwise circles as one looks down the axis from the top of the page
C. toward the top of the page
D. toward the bottom of the page
E. in no direction since B = 0

4. Solenoid 2 has twice the radius and six times the number of turns per unit length as solenoid 1. The ratio of the magnetic field in the interior of 2 to that in the interior of 1 is:
A. 2 B. 4
C. 6 D. 1
E. 1/3

5. A solenoid is 3.0 cm long and has a radius of 0.50 cm. It is wrapped with 500 turns of wire carrying a current of 2.0A. The magnetic field at the center of the solenoid is:
A. 9.9×10^{-8} T B. 1.3×10^{-3} T
C. 4.2×10^{-2} TD. 16T
E. 20T

6. A toroid with a square cross section carries current i. The magnetic field has its largest magnitude:
A. at the center of the hole
B. just inside the toroid at its inner surface
C. just inside the toroid at its outer surface
D. at any point inside (the field is uniform)
E. none of the above

7. A toroid has a square cross section with the length of an edge equal to the radius of the inner surface. The ratio of the magnitude of the magnetic field at the inner surface to the magnitude of the field at the outer surface is:
A. 1/4 B. 1/2
C. 1 D. 2
E. 4

8. A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid?

9. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of B inside the solenoid near its centre.

10. A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid, (b) inside the core of the toroid, and (c) in the empty space surrounded by the toroid.